Problem Set 6 – Due Monday, November 11, 2008

1. Let $f(x) = x \lg x$ (the log being base-2). Compute $f^{-1}(10)$ to at least 4 decimal digits of accuracy. You can do this with the help of a calculator or a short computer program.

2. Sort the following functions into groups $G_1, G_2, \ldots$ such that $f, g \in G_i$ if $f \in \Theta(g)$, and $f \in G_i$ implies that $f \in O(g)$ for each $g \in G_i$. (That is, the slowest growing functions are in the first group; then the next slowest growing functions; and so forth.)

$5n \lg n$	$6n^2 - 3n + 7$	1.5^n	$\lg n^4$	13463
$-15n$	$\lg \lg n$	$9n^{0.7}$	$n!$	$n + \lg n$
$\sqrt{n} + 12n$	$\lg n!$	$\log n$	e^n	2^n

3. Compute the $\Theta(\cdot)$-running time for the following code fragment. Assume that `S` takes unit time to run.

```
for i = 1 to n do
  for j = 1 to i do
    for  k = 1 to 100 do
       for m = j to j+10 do
          S
```

4. Is the following statement true or false? Give a proof or counterexample.

   for every pair of functions f and g, either $f \in O(g)$ or $g \in O(f)$.

5. Prove that if $f_1 \in \Theta(g)$ and $f_2 \in \Theta(g)$ then $f_1 + f_2 \in \Theta(g)$.

6. Determine, with justification, whether each of the following sets is finite, countably infinite, or uncountable:

   (a) $\mathbb{R} \setminus \mathbb{Q}$

   (b) $3\mathbb{Z} - 2\mathbb{Z}$ (where $i\mathbb{Z}$ denotes the set of all integral multiples of i)

   (c) $\{0,1\}^*$, the set of all strings over $\{0,1\}$

   (d) The set of all languages over $\{0,1\}$