Problem Set 7 – Due Monday, November 24, 2008

1. Find the minimum number of students needed to guarantee that 3 of them were born on the same day of the week.

2. Let $A = \{a_1, \ldots, a_7\}$ consist of seven distinct integers. Show that some distinct $x, y \in A$ have a sum or difference divisible by 10.

3. Prove that, for every $n \geq 1$, it is the case that $n^3 + 2n$ is divisible by 3.

4. Prove that, for every $n \geq 1$, it is the case that $(1 + 1/2)^n \geq 1 + n/2$.

5. Prove that for any integer $n \geq 1$, if $x_1, \ldots, x_n$ are distinct real numbers, then no matter how the parentheses are inserted into their product, the number of multiplications used to compute the product is $n - 1$.

6. For $n \geq 1$, let $B(n)$ be the number of ways to express n as the sum of 1s and 2s, taking order into account. Thus $B(4) = 5$ because $5 = 1 + 1 + 1 + 1 = 1 + 1 + 2 = 1 + 2 + 1 = 2 + 1 + 1 = 2 + 2$.

   (a) *Compute* $B(i)$ *for* $1 \leq i \leq 5$ *by showing all the different ways to write these numbers as above.*

   (b) *Find a recursive definition for* $B(n)$ *and identify this sequence.*

   (c) *Compute* $B(10)$.

7. Solve the following recurrence relation by the method of repeated substitution: $T(n) = 3T(n-2)+1$ if $n \geq 2$; and $T(0) = T(1) = 0$. Given an exact answer, not an asymptotic (meaning a big-O) answer.

8. Solve the following recurrence relation to within a $\Theta(\cdot)$ result. Assume that $T(n) \in \Theta(1)$ for sufficiently small n. Use repeated substitution to get your answer. Then compare your answer using the Master Theorem we stated in class. The recurrence is: $T(n) = 2T(n/5) + n$.

9. Same instructions as the last problem; the recurrence is $T(n) = 5T(n/2) + n$.

10. Same instructions as the last problem; the recurrence is $T(n) = 5T(n/5) + n$.