

## Problem Set 8 – Due Tuesday, 9 AM, December 2, 2008

**Note the unusual due date. I will collect problems sets at the beginning of class on Tuesday. Do not turn in your solutions in the box in Kemper.**

*In each of the problems below, if you are calculating a number, give it to me explicitly as a decimal number. You can use Google as a calculator, or any other calculator or program.*

- Given an equal arm balance capable of determining only relative weights of two quantities, and eight coins, all of equal weight except possibly one that may be lighter, explain how to determine if there is a light coin, and how to identify it in just two weighings.
  - Given an equal arm balance as in (a) and  $3^n - 1$  coins,  $n \geq 1$ , all of equal weight except possibly one that is lighter, show how to determine if there is a light coin and how to identify it with at most  $n$  weighings.
- There are 65 students in a class learning how to count. One after another, five students quietly slip out the back door. In how many ways can this exodus occur? (The order in which students leaves does matter.)
- In how many ways can ten boys and four girls sit in a row?
  - In how many ways can they sit in a row if the boys are to sit together and the girls are to sit together?
  - In how many ways can they sit in a row if the girls are to sit together?
  - In how many ways can they sit in a row if *just* the girls are to sit together?
- Four cats and five mice enter a race. The mice are clearly superior; they place first, second, and third. In how many ways can this happen?
- A group of people is comprised of six from the USA, seven from China, and eight from India.
  - In how many ways can a committee of six be formed with two people from each country?
  - In how many ways can a committee of seven be formed with at least two people in each country?
- Eve has ten apples, and plans to give at most three of them to Adam. How many ways can she do this?
- How many five-card hands dealt from a standard deck of 52 playing cards are all of the same suit? I you deal out a random hand, what is the probability that it will have this property?
- How many functions are there from 8-bit strings to 8-bit strings? On this, you do not need to write your answer out in decimal.
  - How many of them are permutations? On this, you do not need to write your answer out in decimal.
- A woman has 9 friends.
  - In how many ways can she invite six of them to dinner?
  - Repeat (a) if two of her friends are divorced (from one another), hate each other, and cannot both be invited.
  - Repeat (a) if the friends consist of three single people and three married couples and if a husband or wife is invited, the spouse must be invited, too.