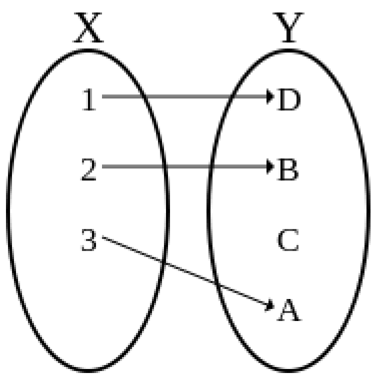
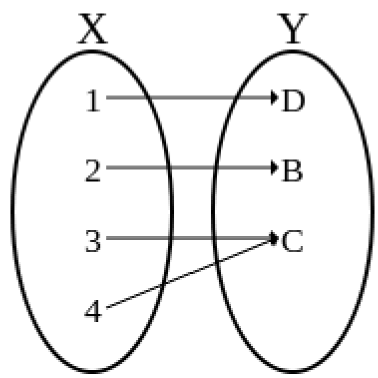
<u>Injective, Surjective, and Bijective functions</u>

A function is **one-to-one** or **injective** if no two distinct elements in the domain maps to the same element in the codomain.

A function is **onto** or **surjective** if every element in the codomain has a pre-image in the domain.

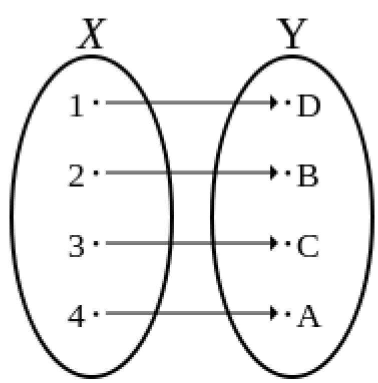


one-to-one (injective),
but not onto



onto (surjective),
but not one-to-one

A function is **bijective** if it is both injective and surjective.



The **identity function** $f(x) = x$ maps an element to itself (from the domain to the codomain). It is useful for many questions in PS6.

<u>PS6 Hints & Notes</u>

2) Consider an infinite set such as $\mathbb{N}$ for counter-examples.

3) Recall that $f^{-1}(5)$ is a value x such that $f(x) = 5$.

4a) Consider defining different maps for different pre-image values (piecewise mapping). To have [0,1] map to (0,1], we have to map 0 to some non-zero image, which forces us to make room for other domain elements. Consider the identity function for certain domain elements.

4b) Consider the identity function. This is a much simpler problem compared to 4a.

5) Recall that the composition of injective maps is injective, and the composition of surjective maps is surjective. (Note that although $\sim$ satisfies the three properties of an equivalence relation, we do not say it is an equivalence relation because there does not exist a set A such that $\sim \subseteq A \times A$.)

6) Consider using contradiction to prove that BIG – Little is uncountable. Consider the fact that a subset of a countable set is still countable.

7) Consider merging two sequences to make one sequence. There are multiple encoding schemes for this question.

8b) Recall that for a set to form a group under an operator $*$

- it must be associative: $(x * y) * z = x * (y * z)$
- there exists an identity 1 such that $x * 1 = 1 * x = x$
- there is always an inverse y such that $x * y = 1 = y * x$