Problem Set 3 – Due Wednesday, October 16, 2013

1. We saw in class how to express any Boolean functionality in **DNF** — disjunctive normal form. Here you will show how to express any Boolean functionality in **CFN** — conjunctive normal form:

   > *A Boolean formula ϕ is in* ***CNF*** *if it is the AND of (one or more) clauses, each clause the OR of (one or more) variables-or-their-complements.*

   For example, $\phi = (A \vee \overline{B})(B \vee \overline{C} \vee \overline{D})(\overline{A} \vee E)$ is in CNF. To illustrate how to express an arbitrary formula in CNF, write a CNF formula for C' and S, as specified by the following truth table. Make sure your method is clear, and that it will work for *any* truth table.

A	B	C	C'
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

A	B	C	S
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

2. Chess is played on an 8×8 board. A *knight* placed on one square can move to any unoccupied square that is at a distance of two squares horizontally and one square vertically, or else two squares vertically and one square horizontally. The complete move therefore looks like the letter L (in some orientation). A knight cannot move off the board. Unlike other chess pieces, the knight can "jump over" other pieces in going to its destination.

   Consider a chess board on which we place any number $m \in \{0, 1, \ldots, 64\}$ of knights, at most one knight on each square. Call the configuration of knights *valid* if no knight can move to a square occupied by another knight.

   Carefully specify a Boolean formula ϕ over 64 Boolean variables $\mathcal{X}$ where the number of truth assignments to ϕ is exactly the number of valid knight configurations.

3. (Velleman, p. 64, problems 8 and 9.) Complete the following table, answering whether the statement is true (True) or false (False) when the universe of discourse is as indicated (the set of reals or the set of integers).

	$\mathbb{R}$	$\mathbb{Z}$
$\forall x \exists y (2x - y = 0)$		
$\exists y \forall x (2x - y = 0)$		
$\forall x \exists y (x - 2y = 0)$		
$\forall x (x < 10 \to \forall y (y < x \to y < 9))$		
$\exists y \exists z (y + z = 100)$		
$\forall x \exists y (y > x \wedge \exists z (y + z = 100))$		

4. (Velleman, p. 72, problem 2.) Translate the negation of the following statements into formulas of quantification logic, introducing predicates as needed.

   (a) There is someone in the freshman class who doesn't have a roommate.

(b) Everyone likes someone, but no one likes everyone.

(c) $(\forall a \in A)(\exists b \in B)(a \in C \leftrightarrow b \in C)$

(d) $(\forall y > 0)(\exists x)(ax^2 + bx + c = y)$

5. Translate the following into a formula of first-order logic. "A language L that is regular will have the following property: there will be some number N (that depends on L) such that if s is a string in L (a *string* is a sequence of characters) whose length is at least N then s can be written as xyz where y is not the empty string and xy^iz is in the language L for every nonnegative integer i." *Note 1: you do not have to understand this statement to do this problem. Note 2: you will see this specific statement again in ECS 120.*