Problem Set 4 – Due Wednesday, October 23, 2013

1. Prove or disprove: for every $N \geq 1$, there is a punctured $N \times N$ grid (that is, an N by N grid with one cell removed) that can be tiled by (L-shaped) trominos.

2. Considered a lucky number, the Thai government decides to issue coins of 9 baht. Show that, for all sufficiently large numbers N you can make N baht using only 9 baht and 10 baht coins.

3. Prove that $n^3 + 2n$ is divisible by 3 for every $n \geq 1$.

4. Prove that $(1 + 1/2)^n \geq 1 + n/2$ for every $n \geq 1$.

5. Prove that $2^n > 10n^2$ for all sufficiently large integers n.

6. Prove that for any integer $n \geq 1$, if $x_1, \ldots, x_n$ are distinct real numbers, then, no matter how the parentheses are inserted into their product, the number of multiplications used to compute the product is $n - 1$.

7. My uncle Joe says he's 1/3 American Indian. When I asked him to explain how that was possible, he told me that his mom and dad were also 1/3 American Indian. Is this a correct proof, by induction, for Joe's possible ancestry?

8. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is false.

   (a) $A \in B \wedge B \in C \implies A \in C$
   (b) $A \subseteq B \wedge B \subseteq C \implies A \subseteq C$
   (c) $A \subsetneq B \wedge B \subsetneq C \implies A \subsetneq C$
   (d) $A \in B \wedge B \subseteq C \implies A \in C$

9. Which of the following conditions imply that $B = C$? In each case, either prove or give a counterexample.

   (a) $A \cup B = A \cup C$
   (b) $A \cap B = A \cap C$
   (c) $A \oplus B = A \oplus C$