Problem Set 5 – Due Tuesday, 4:15 pm, October 29, 2013

Note the unusual due date. I am shifting it up a day to better separate this due date from Thursday's midterm.

1. Write a regular expression for (a) the language A of binary strings whose length is divisible by three; (b) the language B of all binary strings that do **not** contain two consecutive 1's.

2. Determine which of the following relations $\sim$ are: reflexive, symmetric, and transitive.

   (a) $x \sim y$ if x and y are people who were born on the same day.

   (b) $x \sim y$ if x and y are strings which contain a common character.

   (c) $x \sim y$ if x and y are people and there exists a country C such that x has been to country C and y has been to country C.

   (d) $x \sim y$ if x and y are real numbers which are equidistant to the origin.

3. For $a, b \in \mathbb{R}$ define $a \sim b$ if $a - b \in \mathbb{Z}$.

   (a) Prove that $\sim$ defines an equivalence relation on $\mathbb{Z}$.

   (b) What is the equivalence class of 5 — meaning, clearly describe the set $[5] = \{y : 5 \sim y\}$. (Here, $[5]$ is just the name of a set.) Similarly: what is the equivalence class of 5.5?

4. Let $f(x) = x \lg x$ (by $\lg x$ we mean $\log_2 x$). Compute $f^{-1}(10)$ to at least three decimal places of accuracy. Explain how you did this. Note: for a function $f : A \to B$ for which there is exactly one point a that maps to each point $b \in B$, $f^{-1}(y)$ means: the unique x such that $f(x) = y$).

   Note: it looks like I wrote this in an old dialect of C; nowadays, eg, declarations combined with initialization appear to be disallowed or deprecated.