

## Problem Set 9 – Due Wednesday, 4:15 pm, December 4, 2013

As you do these problem, when you get a number that isn't too huge (ten or so digits), please give it to me as an explicit number. If it is really huge, please give an approximation in scientific notation. Being explicit is fun, makes things easier to grade, and lets you exercise that fancy calculator of yours.

1. How many one-to-one functions are there from 4-byte words to 4-byte words? How many onto functions?
2. There are 160 students in a class learning how to count. During the final 10 minutes, one after another, five students quietly slip out the back door (don't think I didn't see). In how many ways can this exodus occur?
3.
  - (a) In how many ways can ten boys and four girls sit down in a row?
  - (b) In how many ways can they sit in a row if the boys sit together and the girls sit together?
  - (c) In how many ways can they sit in a row if the girls are to sit together?
  - (d) In how many ways can they sit in a row if the girls sit together and the boys surround them (annoying creatures that they are)?
4. Four cats and five mice enter a race. The mice are clearly superior; they place first, second, and third. In how many ways can this happen?
5. How many permutations over the letters **a, b, c, d, e, f, g** contain neither the pattern **bge** nor the pattern **eaf**?
6. (Leading zeros are not permitted in either part of this question)
  - (a) How many seven-digit numbers have no repeated digits?
  - (b) How many seven-digit numbers with no repeated digits contain a 3 but not a 6?
7. A group of people is comprised of six from Nebraska, seven from Idaho, and eight from Louisiana.
  - (a) In how many ways can a committee of six be formed with two people from each state?
  - (b) In how many ways can a committee of seven be formed with at least two people in each state?
8. Eve has ten apples, and plans to give at most three of them to Adam. How many ways can she do this?
9. How many five-card hands dealt from a standard deck of 52 playing cards are all of the same suit? I you deal out a random hand, what is the probability that it will have this property?
10. A woman has 9 close friends.
  - (a) In how many ways can she invite six of them to dinner?
  - (b) Repeat (a) if two of her friends are divorced, hate each other, and cannot both be invited.
  - (c) Repeat (a) if the friends consist of three single people and three married couples and if a husband or wife is invited, the spouse must be invited, too.

11. Prof. Rogaway asks a true/false question with ten individual questions. Suppose the ornery professor assigns grades of: 10 points for each correct response, 0 points for each absent response, and  $-10$  points for each incorrect response, but where negative totals are always replaced by zero. Beiter Guess, a student who understands nothing of Rogaway's class, flips a fair coin to decide if he will answer each question true or false.
- (a) In how many different ways might Beiter answer the question?
  - (b) What is the probability that Beiter Guess will earn  $n$  points? (This is just the number of ways to answer the question that will result in  $n$  points divided by the total number of ways of answering the question.)
  - (c) What is the *expected* number of points that Beiter Guess will earn? This is just the sum, over all possible number of points, of that number of points times the the probability of earning it.
  - (d) Should Beiter Guess guess?