Problem Set 9 — Due Wednesday, November 24, at 5pm

1. **(a)** Given an equal arm balance capable of determining only relative weights of two quantities, and given eight coins, all of equal weight except possibly one that is lighter, explain how to determine if there is a light coin, and how to identify it, in just **two** weighings.

   (b) Given an equal arm balance as in (a), and given $3^n - 1$ coins, $n \geq 1$, all of equal weight except possibly one that is lighter, show how to determine if there is a light coin and how to identify it with at most n weighings.

2. Sandy's Sorbet, a small shop on a beach in Phuket, sells $d = 4$ flavors of sorbet, each cleverly named by a number in $[1..d]$. Sandy is willing to put in each cup one and only one flavor. In how many combinations can Ajarn Phil buy $n = 10$ cups of sorbet for the students who join him at the beach? Note that order is be irrelevant here.

   For developing your solution, let $B(d, n)$ be the number of ways to put flavors from $[1..d]$ into the n cups.

3. Sort the following functions into groups $G_1, G_2, \ldots$ such that all functions in a group have the same $\Theta(\cdot)$-complexity, and functions grow asymptotically faster as the group index increases.

$5n \lg n$	$6n^2 - 3n + 7$	1.5^n	$\lg n^4$	$10^{10^{10}}$	$\sqrt{n}$
$15n$	$\lg \lg n$	$9n^{0.7}$	$n!$	$n + \lg n$	$\lg^4 n$
$\sqrt{n} + 12n$	$\lg n!$	$\log n$	e^n	2^n	$n\lceil \lg n \rceil$

4. Compute the $\Theta(\cdot)$-running time for the following code fragment. Assume that `S` takes unit time to run.

```
for i = 1 to n do
  for j = 1 to i do
    for  k = 1 to j*j do
       for m = k to k+100 do
          S
```

5. Solve the following recurrence relations to get at $\Theta(g(n))$ result. Assume that all of the recurrence relations are a positive constant for all sufficiently small n. Show all of your work, not making use of any "Master" theorem you might have seen.

   (a) $T(n) = T(n-1) + n^2$.

   (b) $T(n) = 5T(n/5) + n$.