Problem Set 6 — Due Friday, May 12, 2000

1. Let U be an arbitrary uncountable set and let C be an arbitrary countable one. Prove that $U - C$ is uncountable.
2. Given any 52 numbers, show that there exist two distinct numbers whose sum or difference is divisible by 100.
3. Compute $\gcd(1575, 231)$.
4. Find numbers x and y such that $1575x + 231y = \gcd(1575, 231)$.
5. Compute $\gcd(12345, 54321)$.
6. Prove that $\gcd(n, n+1) = 1$ for all $n \in \mathsf{N}$.
7. A number $n \geq 2$ is called a *2-pseudoprime* if it is composite but $2^{n-1} \equiv 1 \pmod{n}$. Find all 2-pseudoprimes less than 10,000.
8. Alice chooses a random 384-bit number and tests to see if it is prime. Estimate the chance that it is prime (about one in what?)
9. What is $(12345 \times 67890 \times 54321 \times 98765) \bmod 17$?