ECS 20 - Disccusion Week 2

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1 What is a set?

A set is a "collection of distinct objects." For example, the set of people in this Zoom room is $ECS20 = \{Franklin, Zane, Calvin, \dots, Prasad\}$. We call the "objects" *members* of the set or *elements* of the set. We write $Zane \in ECS20$, to mean that Zane is a member of the set ECS20. It is also true that $Franklin \in ECS20$ and $Prasasd \in ECS20$.

The order of the elements in a set does not matter. Each element must be distinct, so no element can appear more than once in a set.

Some common sets in math include the natural numbers

$$\mathbb{N} = \{0, 1, 2, \dots\}$$

and the integers

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}.$$

Another way to describe a set is the following.

$$ECS20 = \{x : x \text{ is in this Zoom room}\}.$$

. The set of even integers is

$$\{x \in \mathbb{Z} : x = 2k, \text{for some integer } k\} = \{\dots, -4, -2, 0, 2, 4, 6, \dots\}$$

2 Rational and Irrational Numbers

Definition: 1) A *rational number* is a number that can be written as p/q, for relatively prime integers p and $q \neq 0$.

2) Two integers p and q are *relatively prime* if they have no common prime factors.

3) An *even integer* n is an integer that can be written $n = 2k$, for some $k \in \mathbb{Z}$.

4) An *odd integer* n is an integer that can be written $n = 2k + 1$, for some $k \in \mathbb{Z}$.

Recall: For a 45-45-90 triangle with legs of length 1, the Pythagorean Theorem tells us that the hypotenuse has length $\sqrt{2}$.

Lemma: Suppose $p \in \mathbb{Z}$. If p^2 is even, then so is p.

Proof of Lemma: We will prove the contrapositive. Suppose p is odd. Then $p = 2k + 1$, for some integer k. Then

$$p^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2 * integer + 1,$$

so p^2 is odd.

$\blacksquare$

Proposition: $\sqrt{2}$ is not rational.

Proof: Suppose, on the contrary, that $\sqrt{2}$ is rational. Then $\sqrt{2} = p/q$, for some relatively prime integers p and q, with $q \neq 0$. Squaring both sides gives $2 = p^2/q^2$, so that $2q^2 = p^2$. Thus, p^2 is even. By the lemma, p is even. So we can write $p = 2k$, for some integer k. We can then write $\sqrt{2} = 2k/q$, so that $2 = 4k^2/q^2$. Then $2q^2 = 4k^2$, and dividing both sides by 2 gives $q^2 = 2k^2$. By the lemma again, q is even. But if p and q are both even, then they are not relatively prime. Therefore, $\sqrt{2}$ is not rational.

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