Midterm

Firstname LASTNAME (Points deducted if not **legible**) Seat#

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For the following questions, please **darken** the correct box. No justification is required.

1. The formula $\phi = (P \vee Q \vee R)$ has exactly *seven* satisfying assignments. (Assume a universe of variables of $\mathcal{U} = \{P, Q, R\}$). **True** **False**
2. It is possible to realize a NAND gate using OR and AND gates. (Recall that the NAND of P and Q is $\neg(P \wedge Q)$.) **True** **False**
3. It is possible to realize an XOR gate using NAND gates. **True** **False**
4. We described in class how every formula ϕ is either *complete* or *sound*. **True** **False**
5. There is a formula ϕ such that ϕ is satisfiable **and** $\neg\phi$ is satisfiable. **True** **False**
6. If A, B, and X are bits, and $X \oplus A = B$, then $X = A \oplus B$. **True** **False**
7. For sets X, Y, and C, if $X \cup C = Y \cup C$ then $X = Y$. **True** **False**
8. For every set A it is the case that $\emptyset \subseteq A$. **True** **False**
9. Regular expressions $(a \cup b)^*$ and $(aa^* \cup b)^*$ denote the same language. **True** **False**
10. Strings under concatenation form a group. **True** **False**
11. Let A and B be sets of strings. Then $A \times B = B \times A$. **True** **False**
12. Let A and B be finite sets. Then $|AB| = |A| \cdot |B|$ **True** **False**

13. Remember the **Towers of Hanoi** problem, where we have n rings on one of three pegs. Using the recursive algorithm described in class, the number of moves T_n needed to transfer these n rings to a different peg is given by the recurrence relation:

$T_0 =$ ____ and $T_n =$ ____ for all $n \geq 1$.

14. Starting at 0, count in **binary** (base-2): ____ ____ ____ ____ ____
Represent your numbers without leading zeros.

15. A **truth table** for $Y = (A \leftrightarrow B) \wedge (B \leftrightarrow C) \wedge (C \leftrightarrow D)$ has 16 rows. It has four columns to specify the input (A, B, C, D) and one column to specify the output (Y). Of the 16 bits that occur in the output, ____ are zero (0) and ____ are one (1).

16. Write a **disjunctive normal form** (**DNF**) formula whose truth table is given below. Your formula should be the **or** of terms where each term is the **and** of variables or their complements:

p	q	r	$F(p,q,r)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

17. **Negate** and simplify the following formula. Your answer should only use addition, exponentiation, $\{\wedge, \vee\}$, and $\{<, \leq, =, \neq, >, \geq\}$.

$$(\exists n)(\exists a)(\exists b)(\exists c)(a^n + b^n = c^n \wedge n \geq 3)$$

Who is this guy?

18. **Translate** the following English sentence into a logical formula:

Some cats can dance—but no such cat can also fetch the morning paper.

The universe $\mathcal{U}$ is "animals." Use predicates of: $C(x)$ for x is cat; $D(x)$ for x can dance; and $F(x)$ for x can fetch the morning paper.

19. Express the following equality in **compact mathematical notation**: Don't use any ellipses (dot-dot-dots, "$\cdots$").

    The sum of the first 100 positive integers is 5050.

20. Express the following as a sentential formula (no quantifiers) in **compact mathematical notation**. Don't use any ellipses. (Hint: You'll want to use a big-$\bigvee$ and/or a big-$\bigwedge$ symbol.)

    At least two of the boolean variables $X_1, \ldots, X_{100}$ are true.

21. We used the **compactness theorem** of sentential logic to show that *what* is true about tiling the plane using a specified set of tile types?

22. To use **mathematical induction** to prove that a proposition $B(n)$ is true for all numbers $n \geq 72$, show that ______ and ______.

23. Explicitly specify the **power set** of the given set:

    $\mathcal{P}(\{\text{big, ghost}\}) =$ ______

24. Let BYTES be the set of 8-bit strings. We defined two addition operations on BYTES that made this set into a *group*: **bitwise-XOR** ($\oplus$) (below left) and (carryless) **computer addition** ($+$) (below right). Add the numbers using each operation.

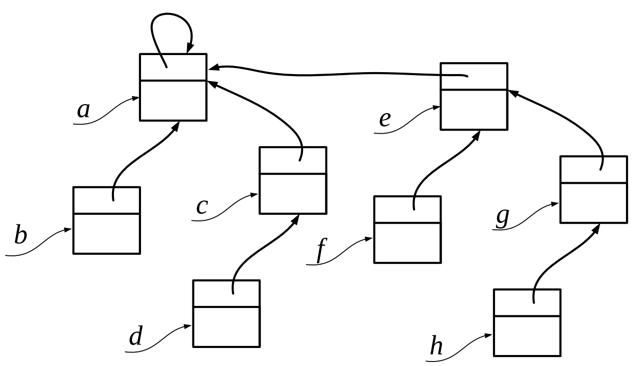
$$\begin{array}{cccccccc c} & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ \oplus & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array} \qquad \begin{array}{cccccccc c} & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ + & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

25. Let $R \subseteq \{a,b\}^* \times \{a,b\}^*$ be the **equivalence relation** defined by $x\,R\,y$ iff the string $|x| = |y|$. Explicitly list the elements of $[aa]$, the block (equivalence class) containing aa.

    $[aa] =$ ______

26. In the Kingdom of Konfusion, coins come in nimes (9¢) and dimes (10¢). Prove that it's possible to make any integral number $n \geq 72$ of cents using only nimes and dimes.

27. If you use the **UNION/FIND** data structure with union-by-rank and collapsing-find, what do you return—and what side effects do you cause—if you call FIND(h) with the following data structure?



28. Write a shortest **regular expression** for the language L that is the set of all binary strings $x \in \{0,1\}^*$ whose length is divisible by three.

29. *Extra credit.* What's an **aardwolf's** favorite food?