Problem Set 3 – Due Wednesday, January 26, at 5 pm

1. Read "A Mathematician's Lament" (Paul Lockhart, 25 pages, 2002) at `https://bit.ly/3fuLNIM` . In a thoughtful essay of 200–400 words (do typeset it, and do include the word count) describe your *personal reaction* to the essay. Your incisive response should make clear that you read and understood Lockhart's critique. Like anything you turn in, make sure your submission is in clear and correct English, and doesn't read like a first draft.

2. Describe the rules of tic-tac-toe as succinctly as you can, but where you feel that no ambiguity remains. Assume your reader is educated and speaks fluent English, but has never heard of tic-tac-toe. How many words did you need?

If you think it ambiguous if something needs to be included in your description, err on the side of concision but then add in a brief explanation as to what the issue is.

3. Complete the following table, answering whether the statement is true (T) or false (F) when the universe of discourse is as indicated (the set of reals, integers, and natural, the last of which includes 0).

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{R}$
$\forall x \exists y (2x - y = 0)$			
$\exists y \forall x (2x - y = 0)$			
$\forall x \exists y (x - 2y = 0)$			
$\forall x (x < 10 \to \forall y (y < x \to y < 9))$			
$\exists y \exists z (y + z = 100)$			
$\forall x \exists y (y > x \wedge \exists z (y + z = 100))$			

4. Translate the following sentence into a statement of first-order logic. The intended universe $\mathcal{U}$ is the set of natural numbers. You may use only the binary operators $+$ and $\cdot$, constants 0 and 1, the binary relation $<$, and the equality symbol.

> Every even number exceeding two is the sum of two primes.

The statement above is known as *Goldbach's conjecture.* After expressing it as a logical formula, mechanically work out the *negation* of the formula, showing your work.

5. Prove that the system of equations:

$$\begin{aligned} 2x + 3y - z &= 5 \\ x - 2y + 3z &= 7 \\ x + 5y - 4z &= 0 \end{aligned}$$

has no solution. We are working here in the real numbers, with arithmetic operations as conventionally understood.

6. Show that $n^2 + n$ is even for any integer n.

7. Prove that $2x^2 - 4x + 3 > 0$ for any real number x.

8. A penny, diameter 0.75 inches, is tossed so as to land at a uniformly random location that falls entirely within the 64 squares of a chess board whose squares are 2-inches by 2-inches. (In a chessboard, squares alternate between black and white.) What is the probability (the chance) that the penny lands entirely within the confines of a white square? This can be worked out with just the most basic intuition about probability.