Problem Set 8 – Due Wednesday, March 9, at 5pm

There are quite a few questions, but most are straightforward. For each numeric answer, please provide it in **two different forms**: (1) a formula involving only constants, $P(n, k)$ values, $C(n, k)$ values, factorials, and basic arithmetic operations; **and** (2) an explicit number, determined by evaluating (1). Being explicit is fun, and also makes things easier to grade.

1. There are 100 students in a class learning how to count. During the last 10 minutes of class, five students quietly slip out the back door (don't think I didn't see). In how many ways can this exodus occur?

2. (a) In how many ways can ten boys and four girls sit down in a row?
   (b) In how many ways can they sit in a row if the boys sit together and the girls sit together?
   (c) In how many ways can they sit in a row if the girls sit together?
   (d) In how many ways can they sit in a row if the girls sit together and the boys surround them (annoying creatures that they are)?

3. Four cats and five mice enter a race. The mice are clearly superior; they place first, second, and third. In how many ways can this happen?

4. How many permutations over the letters `a`, `b`, `c`, `d`, `e`, `f`, `g` contain neither the pattern `bge` nor the pattern `eaf`?

5. (Leading zeros are not permitted in either part of this question)
   (a) How many seven-digit numbers have no repeated digits?
   (b) How many seven-digit numbers with no repeated digits contain a 3 but not a 6?

6. A group of people is comprised of six from California, seven from Oregon, and eight from Washington.
   (a) In how many ways can a committee of six be formed with two people from each state?
   (b) In how many ways can a committee of seven be formed with at least two people from each state?

7. Eve has ten apples, and plans to give at most three of them to Adam. How many ways can she do this?

8. How many five-card hands dealt from a standard deck of 52 playing cards are all of the same suit? If you deal out a random hand, what is the probability that it will have this property?

9. A woman has nine close friends.
   (a) In how many ways can she invite six of them to dinner?
   (b) Repeat (a) if two of her friends are divorced from one another and mustn't both be invited.
   (c) Repeat (a) if the friends consist of three single people and three married couples and if a husband or wife is invited, the spouse must be invited, too.

10. Raccoon walks from $A = (0, 0)$ to $B = (7, 4)$ by going either one unit right (incrementing the first component) or one unit up (incrementing the second component) at each grid point. All such paths are are equally likely. What is the probability that, on raccoon's journey, it passes its uncle's home at $U = (2, 2)$?

11. Raccoon walks from $A = (0, 0)$ by going either one unit right or one up at each grid point. Which of the two determined by flipping a fair coin. What is the probability raccoon passes both $U = (2, 2)$ and $B = (7, 4)$ along its journey?

12. Prof. Hardy asks ten true/false questions. The ornery professor assigns grades of: 10 points for each correct response, 0 points for each absent response, and -10 points for each incorrect response—but where negative totals are always replaced by zero. Poor Ziggy understands nothing of Hardy's class, so flips a fair coin to decide if he will answer each question true or false (he leaves no question unanswered).

    (a) For each number of points N, what is the probability that Ziggy will earn N points?

    (b) What is the expected number of points $E[X]$ that Ziggy will earn?

    (c) Is Ziggy wise to guess (as opposed to leaving things blank)?

13. Are the following two graphs isomorphic? Prove your answer.