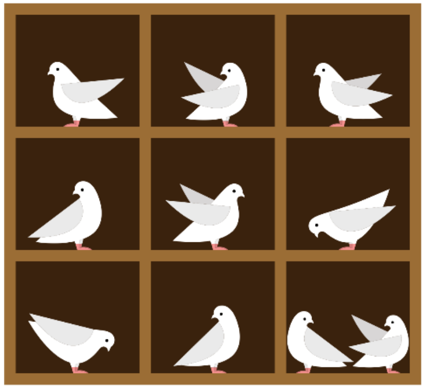
Lecture 5 (3T)

Proofs



Announcements

- Quiz 1 performance wasn’t bad
- Zane to review of logic next week in discussion

Today:

▢ Proofs illustrating sundry techniques

Phillip Rogaway

Problem-Solving Techniques

From Lecture 1

1. Reformulate to something equivalent
2. Generalize
3. Work out special cases. Small cases. Look for patterns.
4. Name things (e.g., introduce variables)
5. Create tailor-made definitions
6. Draw pictures
7. Think recursively
8. Adopt a playful attitude
9. Forget pattern-matching
10. But look for echoes
11. Know what you know (don't fool yourself, don't try to fool others)
12. Give serious attention to exposition. Never turn in a first draft.
    Critically read what you write.

- 2. Proofs
  - 2.1 Mathematical definitions
  - 2.2 Introduction to proofs
  - 2.3 Best practices and common errors in proofs
  - 2.4 Writing direct proofs
  - 2.5 Proof by contrapositive
  - 2.6 Proof by contradiction
  - 2.7 Proof by cases

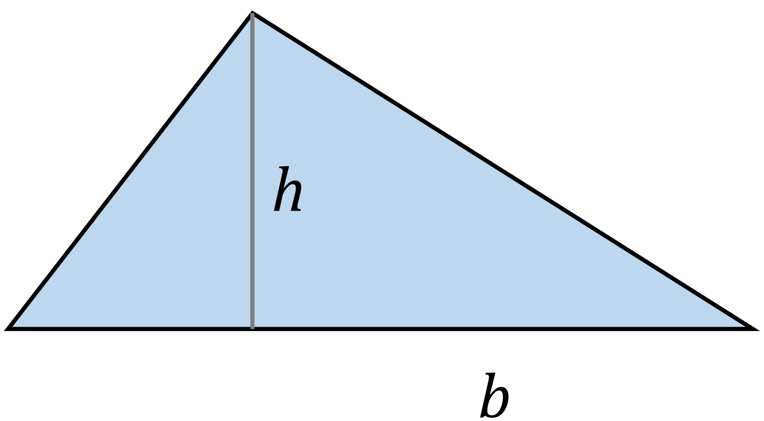
- Follow-your-nose proof
- Introduce-the-right-extra-thing proof
- Draw-the-right-picture proof
- Find-a-good-counterexample proof
- Find-a-good-representation proof
- Break-into-good-cases proof
- Reduce-to-known-result proof

Example 1: $\sqrt{2}$ is irrational.

Example 2: There are **irrational** numbers a and b such that a^b is **rational**.

Example 3: In playing tic-tac-toe, if the first player moves to a corner then the second player must take the center, or else the first player can force a win.

Example 4: The area of a triangle with three acute angles is $bh/2$ where b is the triangle's **base** and h is its height.



Example 5: $1 + 2 + \cdots + n =$

Example 6: $1 + 3 + 5 + \cdots + 2n - 1 =$

Example 7:

20 random cards are placed in a row, all face **down**. A **move** consists of turning a face-down card face-up and turning over the card, if any, immediately to the **right**. Show that no matter what the choice of cards to turn, this sequence of moves **must** terminate.

Example 8: [MIT book, chapter 11]

Who, on average, has more opposite-gender partners: men or women? …

In one of the largest [studies], researchers from the University of Chicago interviewed a random sample of 2500 people over several years … Their study, published in 1994, … found that **men have on average 74% more opposite-gender partners than women**.

Other studies have found that the disparity is even larger. In particular, ABC News claimed that **the average man has 20 partners over his lifetime**, and **the average woman has 6**, for **a percentage disparity of 233%.** The ABC News study [claimed] a **2.5% margin of error**…

Proposition: In a **bipartite** graph G=(V_1, V_2, E) with m **edges** and n **vertices** on each side of the bipartition, the average **degree** of a vertex in V_1 is the same as the average degree of a vertex in V_2.

Example 9:

Show that any party with 6 people will contain a group of 3 mutual friends or a group of 3 mutual non-friends.

Example 10: The last theorem wouldn't work for 5 people.

Namely: you **could** have a party with **five** people and where **no three** are mutual **friends** and **no three** are mutual **non-friends**.

Example 11: Show that $\{\longrightarrow, \oplus\}$ is functionally complete.

Some Truths about Proofs

1. Finding proofs is **not** mechanical; it is an **art**.
2. Mathematical discovery is **more than proofs**:
   guessing and discovering results.
   Only in a class setting do proofs come as “prepackaged” task
3. Don’t get so obsessed with rigor that you fail to develop and refine intuition – and to **err**.
4. Proofs **evolve.** They can be quite **dialectical.**
5. Intuition can be **lost** in a refined, succinct proof.
   Proofs are not “born” in such a manner
6. You can’t prove what doesn’t **make sense** to you.
   Don’t even try to prove something until you get to the point of the language and claim making sense.