Problems 9–12. Due Thursday, May 21, 1998

Problem 9. Consider the following INEQUIVALENCE REGULAR EXPRESSIONS problem:

Instance: Regular expressions α and α'.
Questions: Is $L(\alpha) \neq L(\alpha')$? Show that INEQUIVALENCE OF REGULAR EXPRESSIONS is NP-Complete, even if the regular expressions do not contain the $*$-operator (that is, they only use union and concatenation).

Problem 10. We have exhibited a polynomial-time algorithm to minimize the number of states in a DFA. Show that there is no polynomial time algorithm for minimizing the number of states in an NFA. Assume P$\neq$NP.

Problem 11. For a language $L \subseteq \Sigma^*$ and a function $f\colon \Sigma^* \to \Sigma^*$ we let

$$L_f = \{ x \in \Sigma^* : f(x) \in L \} .$$

We say that a complexity class is *closed under composition* if for any L in the class and any polynomial time computable function f it is the case that the language L_f is also in the class.

For a language A let $A^{\#} = \{ x \,.\, 0^{|x|^2 - |x|} : x \in A \}$ be the language consisting of elements x of A appended with $k^2 - k$ zeros, where $k = |x|$.

(a) Show that **NP** is closed under composition.

(b) Show that if $A \in \textbf{SPACE}(n^2)$ then $A^{\#} \in \textbf{SPACE}(n)$.

(c) Show that $\textbf{SPACE}(n)$ is *not* closed under composition.

(d) Conclude that $\textbf{NP} \neq \textbf{SPACE}(n)$.

Problem 12. Sipser, p. 303, Problem 8.14.