Problems 12–15. Due Thursday, June 4, 1998

Problem 12. Sipser, p. 303, Problem 8.14. *Delayed from last assignment.*

Problem 13. Show that 2SAT is NL-complete.

Hint: Use that NL *is closed under complement, and that* PATH *is* NL*-complete even for a DAGs (why?). Showing membership in* NL *is the harder part. Think of a clause* $(x \vee y)$ *as indicating that* $\overline{x} \Rightarrow y$ *and* $\overline{y} \Rightarrow x$. *Given a* 2SAT *instance* ϕ *construct a graph* G *whose vertex set is the variables and their complements. Put in the right edges and (with proof) relate some graph-theoretic property of* G *to the satisfiability of* ϕ.

Problem 14. By P^A we mean the class of languages decidable in polynomial time given an A oracle. By NP^A we mean the class of languages verifiable in polynomial time given an A oracle. Show that $\mathrm{P}^{\mathrm{TQBF}} = \mathrm{NP}^{\mathrm{TQBF}}$.

Problem 15. Show that if NP$\subseteq$BPP then NP=RP. (Recall that $L \in$RP means that there exists a PPT TM M such that, when $x \in L$, $\Pr[M(x) = 1] \geq 1/2$; and when $x \notin L$, $\Pr[M(x) = 1] = 0$.)

Hint: Use that self-reducibility of SAT.