

## Problem Set 4: Problems 12–15. March 2, 2006

**Problem 12.** Problem 9.5.29 (a).

**Problem 13.** Problem 11.5.18.

**Problem 14.** Problem 11.5.23.

**Problem 15.** (*The following is Problem 8.15 from Sipser (2nd edition).*) The cat-and-mouse game is played by two players, “Cat” and “Mouse,” on an arbitrary undirected graph. At a given point each player occupies a node of the graph. The players take turns moving to a node adjacent to the one that they currently occupy. A special node of the graph is called “Hole.” Cat wins if the two players ever occupy the same node. Mouse wins if it reaches the Hole before the preceding happens. The game is a draw if a situation repeats (i.e., the two players simultaneously occupy positions that they simultaneously occupied previously and it is the same player’s turn to move). Define  $HAPPY-CAT = \{\langle G, c, m, h \rangle \mid G, c, m, h \text{ are respectively a graph, and positions of the Cat, Mouse, and Hole, such that Cat has a winning strategy if Cat moves first}\}$ . Show that  $HAPPY-CAT$  is in P. *Hint: The solution is not complicated and doesn’t depend on subtle details in the way the game is defined. Consider the game tree. It is exponentially big, but you can search it in polynomial time.*