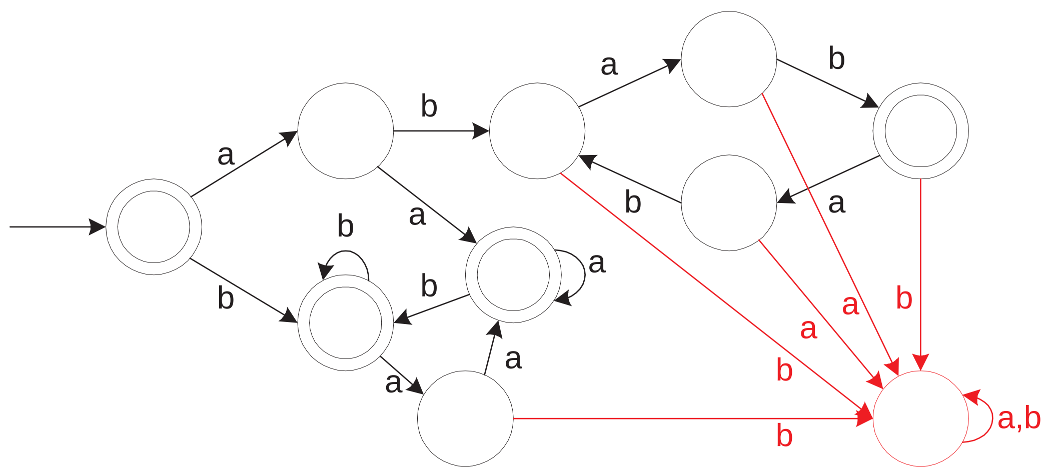
ECS 120: Theory of Computation

Homework 4 Solution

Due: 4/26/06

Problem 1.

[Linz, Section 3.2, Exercise 4c).]



[Linz, Section 3.2, Exercise 9.]

$L((a^+(a+b)^+ + a^*(a^*b+c))(a+b^*)^*)$

[Linz, Section 3.2, Exercise 15.]

$(+ + - + \lambda)\cdot$
$(0+1+2+3+4+5+6+7+8+9)\cdot$
$(0+1+2+3+4+5+6+7+8+9)^*\cdot$
$(0+1+2+3+4+5+6+7+8+9)^*\cdot$
$(\lambda + (E(0+1+2+3+4+5+6+7+8+9)*)$

[Linz, Section 3.3, Exercise 6.]

$S \rightarrow aaA|\lambda$
$A \rightarrow bA|abS$

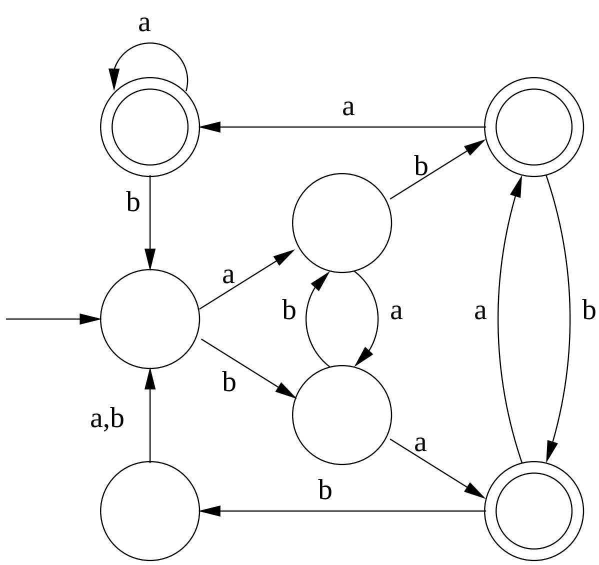
[Linz, Section 3.3, Exercise 10.]

$S \rightarrow Aab|\lambda$
$A \rightarrow Ab|Saa$

[Linz, Section 3.3, Exercise 14.]

Let L be a regular language accepted by a dfa $M = (Q, \Sigma, \delta, q_0, F)$.
Since M does not contain λ, $q_0 \notin F$.
Then L can be defined by a right-linear grammar $G = (Q, \Sigma, q_0, P)$, where P is defined as the following:
$q_k \rightarrow aq_l$, where $q_k, q_l \in Q$, $a \in \Sigma$, and $\delta(q_k, a) = q_l$
$q_k \rightarrow a$, where $q_k \in Q$, $a \in \Sigma$, and $\delta(q_k, a) \in F$.

Problem 2.



Problem 3.

Construction of the DFA:
Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we construct $\widehat{M}$ that accepts $chop(L(M))$.
$\widehat{M} = (Q, \Sigma, \delta, q_0, \widehat{F})$, where $\widehat{F} = \{q \in Q : \exists a \in \Sigma \text{ for which } \delta(q, a) \in F\}$.

Show that $L(\widehat{M}) = chop(L(M))$ by showing $x \in L(\widehat{M}) \Longleftrightarrow x \in chop(L(M))$:

If $w \in L(\widehat{M})$, then $\delta^*(q_0, w) \in \widehat{F}$. That means there should exist an $a \in \Sigma$ such that $\delta^*(q_0, wa) \in F$ and $wa \in L(M)$. Thus, $w \in chop(L(M))$ and $L(\widehat{M}) \subset chop(L(M))$.

If $w \in chop(L(M))$, then $\exists a \in \Sigma$ such that $wa \in L(M)$. That means $\delta(\delta^*(q_0, w), a) \in F$ and $\delta(q_0, w) \in \widehat{F}$. Thus, $w \in L(\widehat{M})$ and $chop(L(M)) \subset L(\widehat{M})$.

Since $L(\widehat{M}) \subset chop(L(M))$ and $chop(L(M)) \subset L(\widehat{M})$, $L(\widehat{M}) = chop(L(M))$.
We have shown that L is regular and that there is a DFA that accpets $chop(L)$.
Thus, regular languages are closed under the *chop* operation.