ECS 120: Theory of Computation

Homework 5 Solution

Due: 5/3/06

Problem 1.

[Linz, Section 4.2, Exercise 6).]

Before illustrating the algorithm, we need to prove that the family of regular languages are closed under reversal.

Let $L^R = \{w : w^R \in L\}$ and prove that L^R is also regular:
Since L is regular, it is accepted by a DFA $M = (Q, \Sigma, \delta, q_0, F)$.
Let $M_R = (Q \cup \{q_R\}, \Sigma, \delta_R, q_R, \{q_0\})$ be an NFA that accpets L^R, where:

- q_R is the new start state. Let $\delta_R(q_R, \lambda) = F$.
- For each transition in δ: $\delta(q_i, a) = q_j \Longrightarrow \delta_R(q_j, a) = q_i$.
- $\{q_0\}$ is the set of final states for M_R.

You can prove by induction that $L(M_R) = L^R$ – i.e., $x \in L(M_R) \Leftrightarrow x \in L^R$.

The following describes an algorithm that determines whether a regular language L contains any string w such that $w^R \in L$ in finite steps:

1. Construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$, where L = L(M).
2. Construct a DFA M_R, where $L(M_R) = \{w : w^R \in L\}$.
3. Coonstruct a DFA M', such that $L(M') = L(M) \cap L(M_R)$ (based on Theorem 4.1).
4. If $L(M') \neq \emptyset$ (using the algorithm from Theorem 4.6 to determine this property), then there exists some $w^R \in L$.

[Linz, Section 4.2, Exercise 14.]

The following describes an algorithm that determines whether a regular language L contains infinite number of even-length strings in finite steps:

1. Construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$, where L = L(M).
2. Construct a DFA M_E, where $L(M_E) = \{w : w \in \Sigma^* \text{ and } |w| \bmod 2 = 0\}$.
3. Coonstruct a DFA M', such that $L(M') = L(M) \cap L(M_E)$ (based on Theorem 4.1).
4. If $L(M')$ is infinite (using the algorithm from Theorem 4.6 to determine this property), then L contains infinite even-length strings.

[Linz, Section 4.3, Exercise 5(d).]

No. Prove by contradition using the pumping lemma:
Given m, let $w = a^{2^m}$, which is in L. w can be decomposed into xyz, where $|xy| \leq m$ and $y \neq \lambda$. Suppose $y = a^k$, where $1 \leq k \leq m$, then we pump i times to generate a string that contains $2^m + k \cdot (i-1)$ a's. Let $i = 2$, then $xy^2z = a^{2^m+k}$. Since $2^m + k < 2^{m+1}$, $a^{2^m+k} \notin L$. Thus, L is not regular.

[Linz, Section 4.3, Exercise 5(e).]

No. Prove by contradition using the pumping lemma:
Given m, let $w = a^{p \cdot q}$, where p and q are prime numbers and $p \cdot q \geq m$. w can be decomposed into xyz, where $|xy| \leq m$ and $y \neq \lambda$. Suppose $y = a^k$, where $1 \leq k \leq m$, then we pump i times to generate a string that contains $p \cdot q + k \cdot (i-1)$ a's. Let $i = 1 + p \cdot q$, then $p \cdot q + k \cdot [(1 + p \cdot q) - 1] = p \cdot q + k \cdot p \cdot q = p \cdot q \cdot (k+1)$. Since $p \cdot q \cdot (k+1)$ cannot be a product of two primes, $a^{p \cdot q \cdot (k+1)} \notin L$. Thus, L is not regular.

[Linz, Section 4.3, Exercise 5(g).]

$L^* = \{a^n : n \geq 2, \text{ is the sum of primes}\} = \{a^n : n = 0 \text{ and } n \geq 2\}$. Since a simple DFA can be constructed for L^*, L^* is regular.

[Linz, Section 4.3, Exercise 10(a).]

Given m, let $w = a^{(m!)^2+1} \in L$. w can be decomposed into xyz, where $|xy| \leq m$ and $y \neq \lambda$. Suppose $y = a^k$, where $1 \leq k \leq m$, then we pump i times to generate a string with $(m!)^2 + 1 + k \cdot (i-1)$ a's. Let $i = 1 + \frac{2 \cdot m!}{k}$.

Then, $(m!)^2 + 1 + k \cdot [(1 + \frac{2 \cdot m!}{k}) - 1] = (m!)^2 + 2(m!) + 1 = (m! + 1)^2$.
Thus, L is not regular.

[Linz, Section 4.3, Exercise 10(b).]

Example 4.11 (on page 119) shows that $\overline{L}$ is not regular. Thus, by closure properties L is not regular.

[Linz, Section 4.3, Exercise 15(e).]

No. Prove by contradiction using the pumping lemma.
Given m, let $w = a^m b^m \in L$. w can be decomposed into xyz, where $|xy| \leq m$ and $y \neq \lambda$. Since $|xy| \leq m$, y contains only a's. Suppose $y = a^k$, where $1 \leq k \leq m$, then we pump i times to generate a string with $m + k \cdot (i-1)$ a's and m b's. Let $i = 2$, then $xy^2z = a^{m+k}b^m$. Since $m < m + k$, $a^{m+k}b^m \notin L$.
Thus, L is not regular.

[Linz, Section 4.3, Exercise 15(f).]

Yes. We can construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts L, where $Q = \{q_i : 0 \leq i \leq 201\}$, $\Sigma = \{a, b\}$, $F = \{q_i : 100 \leq i \leq 200\}$, and δ is defined as follows:

- $\delta(q_{201}, a) = q_{201}$ and $\delta(q_{201}, b) = q_{201}$
- $\delta(q_j, a) = q_{j+1}$ and $\delta(q_j, b) = q_{201}$, for $0 \leq j < 100$
- $\delta(q_{100}, a) = q_{100}$
- $\delta(q_j, b) = q_{j+1}$ and $\delta(q_j, a) = q_{201}$, for $100 \leq j < 200$
- $\delta(q_{200}, a) = q_{201}$ and $\delta(q_{200}, b) = q_{201}$

[Linz, Section 4.3, Exercise 24.]

No. For example, suppose $L_1 = L(a^*b^*)$ and $L_2 = \{a^n b^n : n \geq 0\}$. Clearly, L_1 is regular, but L_2 is not (shown in Example 4.7). However, $L_1 \cup L_2 = L(a^*b^*)$ is regular.

[Linz, Section 4.3, Exercise 26.]

L is regular and can be accepted by a DFA similar to Section 4.3, Exercise 15(f).

[a.]

No. The pumping lemma is used for proof by contradiction. Although we could show that any pumped string is still in L, there is nothing in the pumping lemma that allows us to conclude that L is regular.

[b.]

No. For any given value of m, there is always a w such that $w_i \in L$ where $i \geq 0$.