ECS 120: Theory of Computation

Homework 6 Solution

Due: 5/10/06

Problem 1.

[Linz, Section 6.2, Exercise 2).]

$S \to AC$
$C \to SB$
$A \to a$
$B \to b$

[Linz, Section 6.2, Exercise 11.]

$S \to aSB$
$A \to a$
$B \to b$

[Linz, Section 7.1, Exercise 4(d).]

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[Linz, Section 7.1, Exercise 4(h).]

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[Linz, Section 7.1, Exercise 4(k).]

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[Linz, Section 7.1, Exercise 6.]

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[Linz, Section 7.1, Exercise 17.]

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$ be an npda, where

$L(M) = \{w \in \Sigma^* : (q_0, w, z) \overset{*}{\vdash}_M (p, \lambda u), p \in F, u \in \Gamma^*\}$.

We construct an npda $\widehat{M}$, such that $N(\widehat{M}) = L(M)$ and $\widehat{M} = (Q \cup \{p_f\}, \Sigma, \Gamma, \widehat{\delta}, q_0, z, \{p_f\})$:

- $\widehat{\delta}(p, \lambda, \gamma) = \{(p, \lambda)\}, p \in F, \gamma \in \Gamma$
- $\widehat{\delta}(p, \lambda, z) = \{(p_f, z)\}, p \in F$

Prove that $x \in L(M)$ iff $x \in N(\widehat{M})$:
If $x \in L(M)$, then $(q_0, x, z) \overset{*}{\vdash}_M (p, \lambda, u), p \in F, u \in \Gamma^*$. When feeding x to $\widehat{M}$, the new transitions will continuously pop out the existing stack symbols until the stack is emtpy. Thus, $(p, \lambda, u) \overset{*}{\vdash}_{\widehat{M}} (p_f, \lambda, z), p \in F, u \in \Gamma^*$ and $x \in N(\widehat{M})$.
If $x \in N(\widehat{M})$, then there must exist $u \in \Gamma^*$ and $p \in F$ such that $(q_0, x, z) \overset{*}{\vdash}_{\widehat{M}}$ (p, λ, u) and $(p, \lambda, u) \overset{*}{\vdash}_{\widehat{M}} (p_f, \lambda, z)$. Since M finish reading x as it reaches a state in F, $x \in L(M)$.
Therefore, $L(M) = N(\widehat{M})$.

Problem 2.

[a.]

$L = \{aaa^n b^n : n \geq 1\}$.

[b.]

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[c.]

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[d.]

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