ECS 120: Theory of Computation

Homework 8 Solution

Date: 5/31/06

[Problem 1.]

[Linz, Section 11.1, Exercise 10]

Yes. Suppose L_1 and L_2 are recursive languages and can be accepted by turing machines M_1 and M_2, respectively. We construct a turing machine $\widehat{M}$, such that:

1. Given an input $w \in \Sigma^*$
2. Break w into two substrings w_1 and w_2
   (a) Run M_1 with w_1 and M_2 with w_2 seperately in parallel
   (b) If $w_1 \in L(M_1)$ and $w_2 \in L(M_2)$, then $\widehat{M}$ accepts w.

Clearly, $\widehat{M}$ accepts $L_1 \cdot L_2$. Also, $\widehat{M}$ can decide $w \in \Sigma^*$ in finite number of steps, because step 2 takes at most $|w| - 1$ iterations, and step $2(a)$ can be done in finite steps (L_1 and L_2 are recursive). Therefore, $L_1 \cdot L_2$ is recursive.

[Linz, Section 11.1, Exercise 16]

Proof by contradiction. Suppose $S_1 - S_2$ is finite, and therefore is countable. Then S_2 must countable, since S_1 is countable. However, S_2 is uncountable, thus $S_2 - S_1$ is infinite and uncountable.

[Linz, Section 11.1, Exercise 19]

Consider numbers between 0 and 1. Irrational numbers have been defined as decimal (non-periodic) fractions. Assume it is possible to enumerate all such decimals. Let's choose an enumeration and list the decimals in the corresponding order:
$a_1 = 0.a_{11}a_{12}a_{13}a_{14}...$
$a_2 = 0.a_{21}a_{22}a_{23}a_{24}...$
$a_3 = 0.a_{31}a_{32}a_{33}a_{34}...$
...

where a_{mn} stands for the n^{th} digit of the m^{th} decimal. Apply Cantor's diagonal process. To remind, we made an assumption that all the decimals between 0 and 1 have been listed in the sequence above. Proof by contradiction by showing that at least one decimal is missing from the list. The decimal $b = 0.b_1b_2...$ is constructed a digit by digit. Select b_1 to be any digit but a_{11}. Select b_2 to be any digit but a_{22}. And in general, select b_n to be any digit but a_{nn}. Then b can't equal any decimal a_n, $n = 1, 2, 3, ...$ because b differs from a_1 in the first digit; it differs from a_2 in the second digit and so on.

[Linz, Section 11.2, Exercise 8]

For each prduction rule $u \to v$, where $|u|, |v| > 2$, rewrite u and v such that $|u|, |v| \leq 2$ and $|u| \leq v$. For example, let us rewrite $aABcC \to aBAc$:

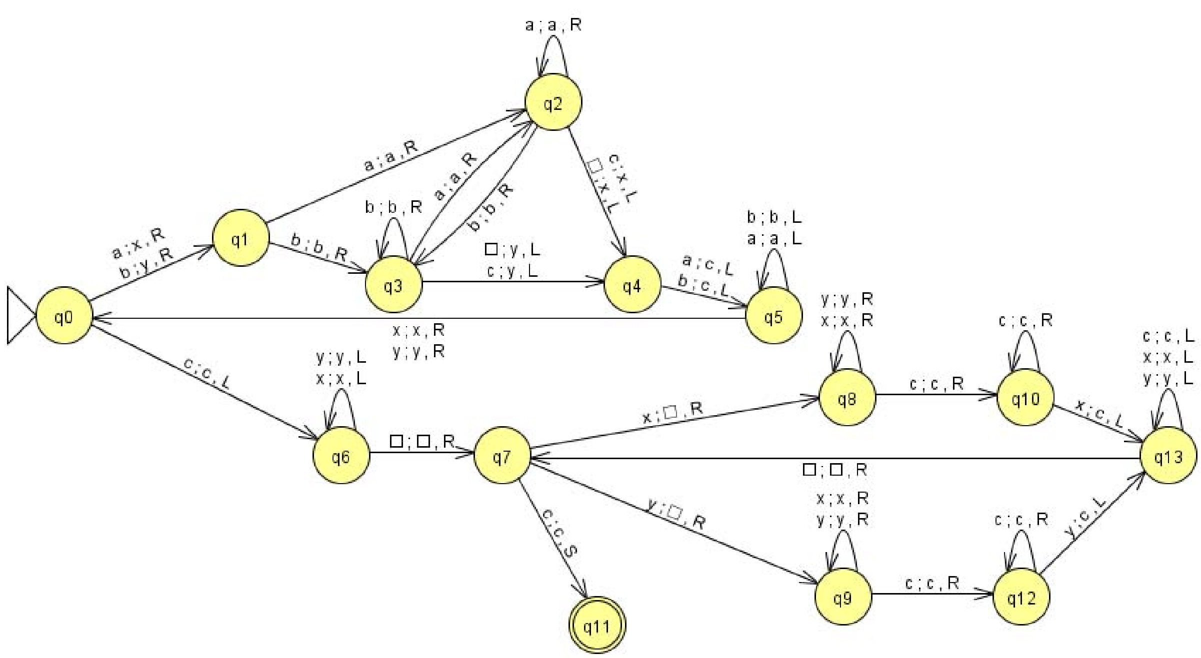
The first step is to rewrite u:
$aB \to V_1V_0$
$V_1c \to V_2V_0$
$V_2C \to aBAc$
$V_0 \to \lambda$

The second step is to rewrite v using similar techniques that convert CFGs to Chomsky normal form:
$V_2 \to V_3V_4$
$V_3 \to aB$
$V_4 \to Ac$

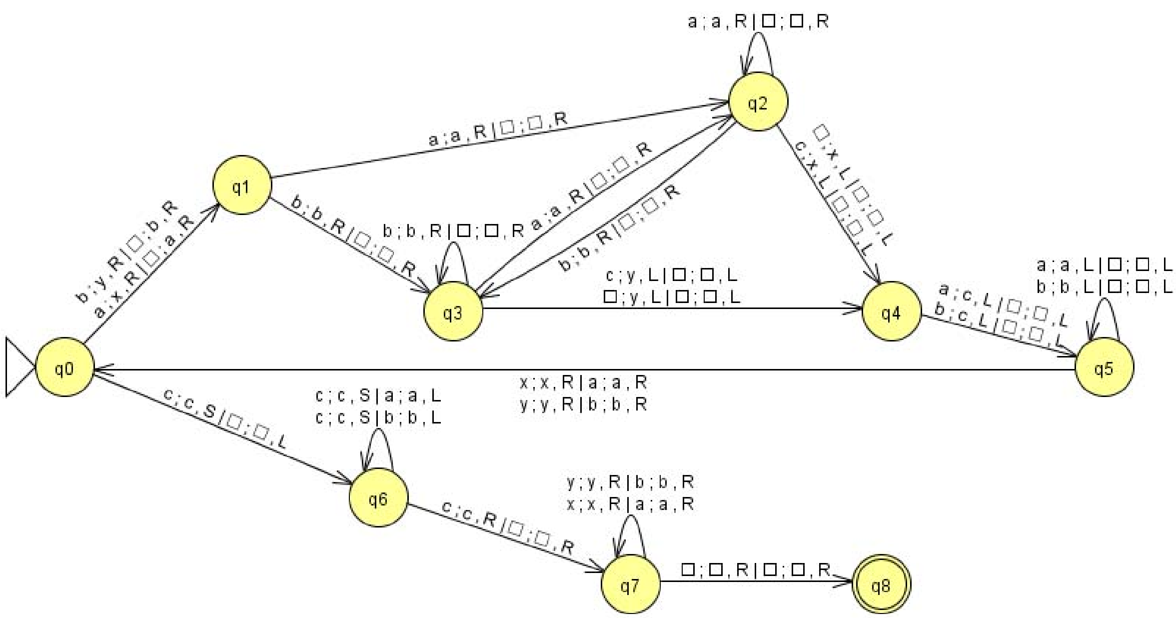
For $u \to v$, where $|u| = 2$ and $|v| = 1$, append V_0 to v as used in Exerise 7.

[Problem 2.]

[a.]



[b.]



[c.]

$T \to aATZ|bBTZ|aa|bb$
$Aa \to aA$
$Ab \to bA$
$Ba \to aB$
$Bb \to bB$
$AZ \to Za$
$BZ \to Zb$
$Z \to \lambda$

Z is used to prevent subsequent swapping.